



Axiomatic Definition of Valid 3D Parcels, potentially in a Space Partition

18-11-2011

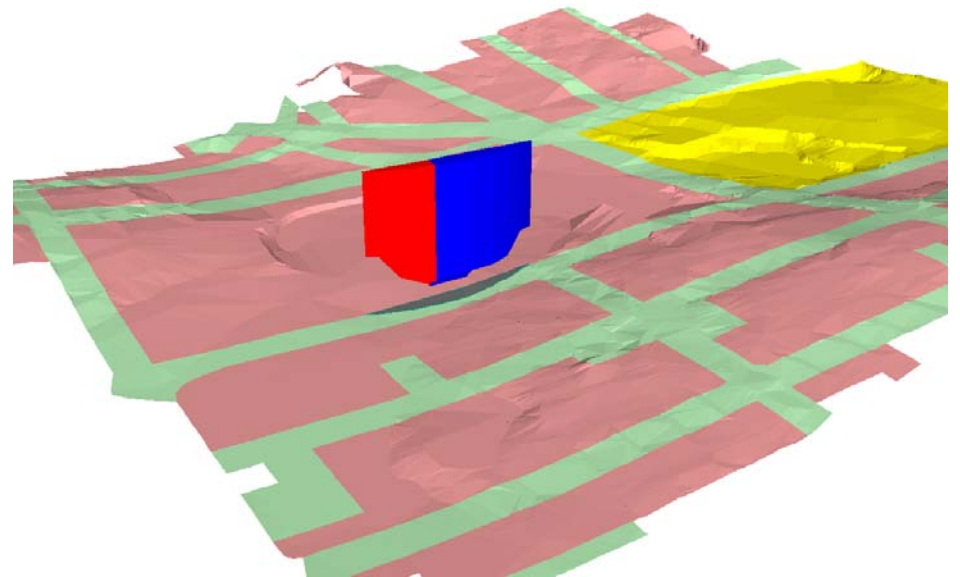
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2nd International Workshop on 3D Cadastres, 16-18 Nov'11, Delft, NL

Contents

- Introduction & background
- Basic Definitions
- Axioms
- LADM
- Other Issues
- Conclusion & future work



The Situation in 2D

(Or, we all know what a polygon is, don't we?)

Original
Geometry
construction

MapInfo

ArcMap

GML (Gaia)

Selects

Draws



both
clockwise

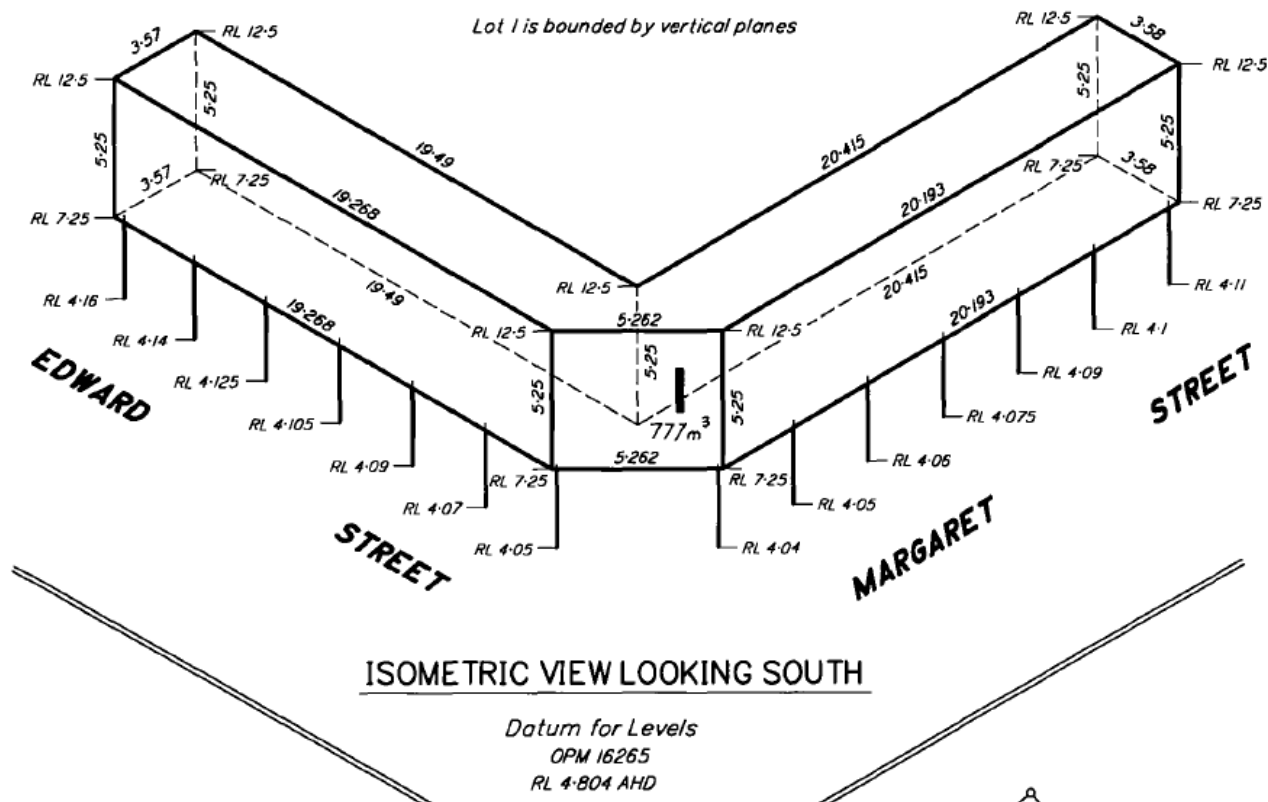
1 clockwise
1 anticlockwise



van Oosterom, P., W. Quak and T. Tijssen (2004). About Invalid, Valid and Clean Polygons. Developments In Spatial Data Handling. P. F. Fisher. New York, Springer-Verlag: 1-16.

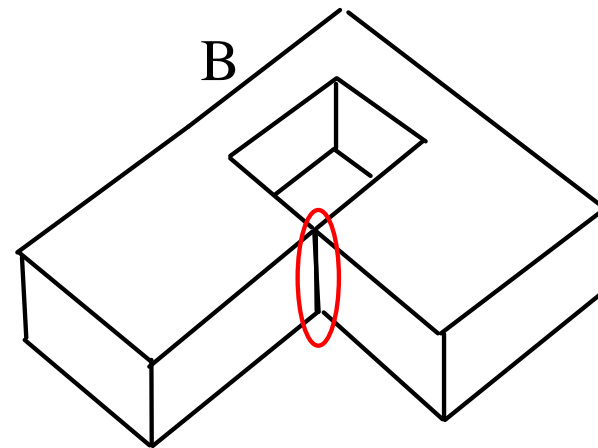
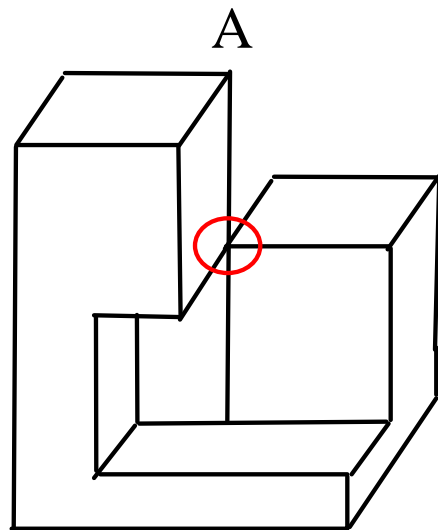


3D Cadastre

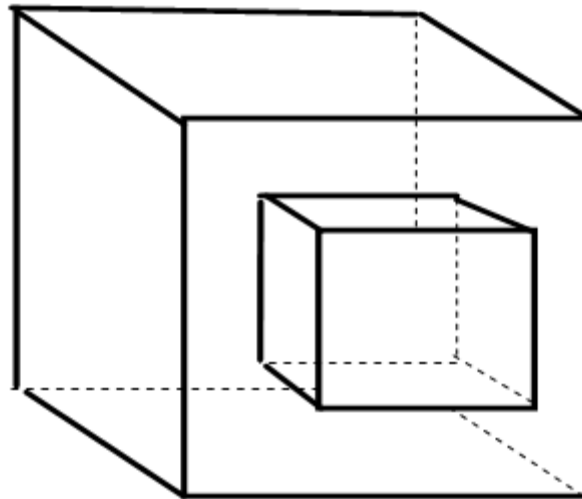


- 3D Parcel: the spatial unit against which (one or more) unique and homogeneous rights responsibilities or restrictions are associated to the whole entity, as included in a Land Administration system (based on ISO 19152) → in practice often a single interior-connected volume

Valid, but non-manifold boundary

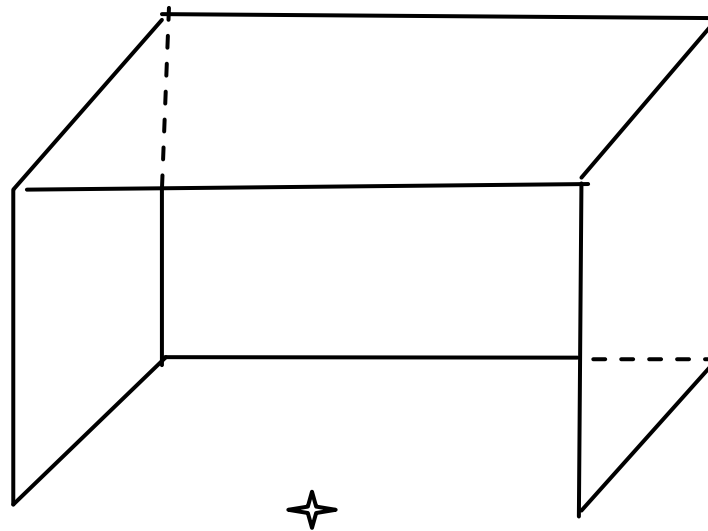


Valid, disallowed by Oracle 3D



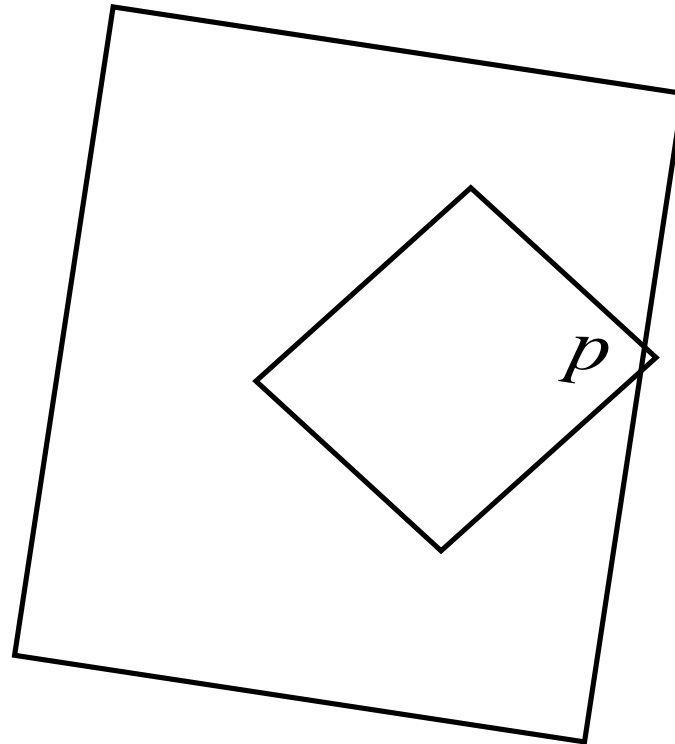
A face with a hole is disallowed

Invalid, can not answer question:
Is point within region?

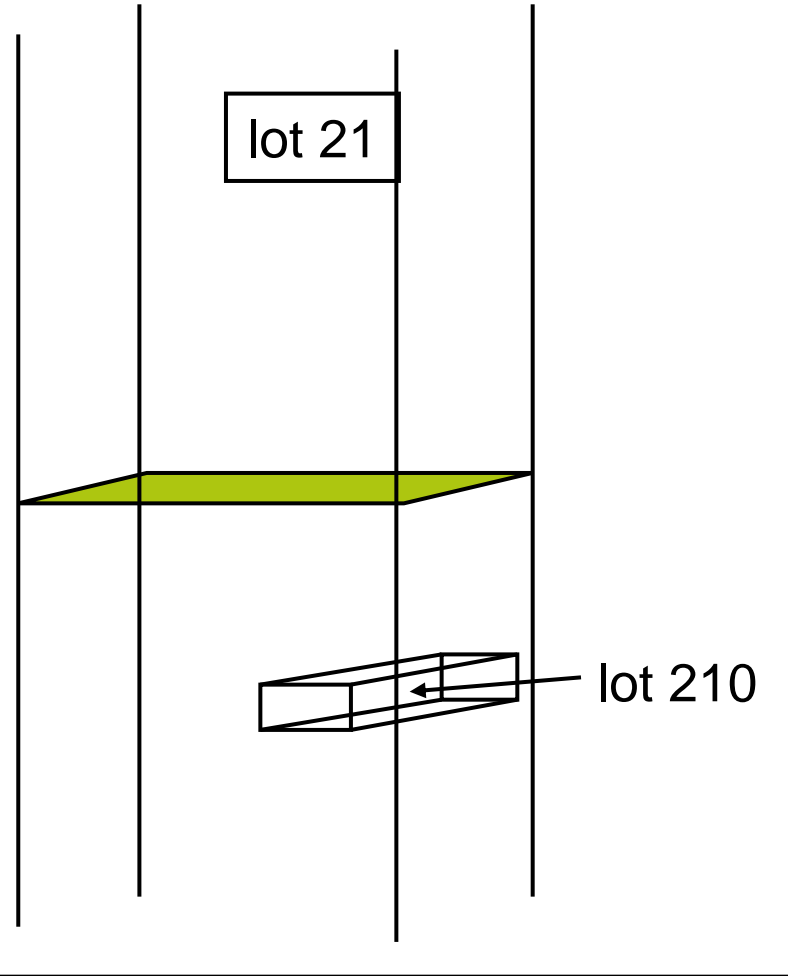
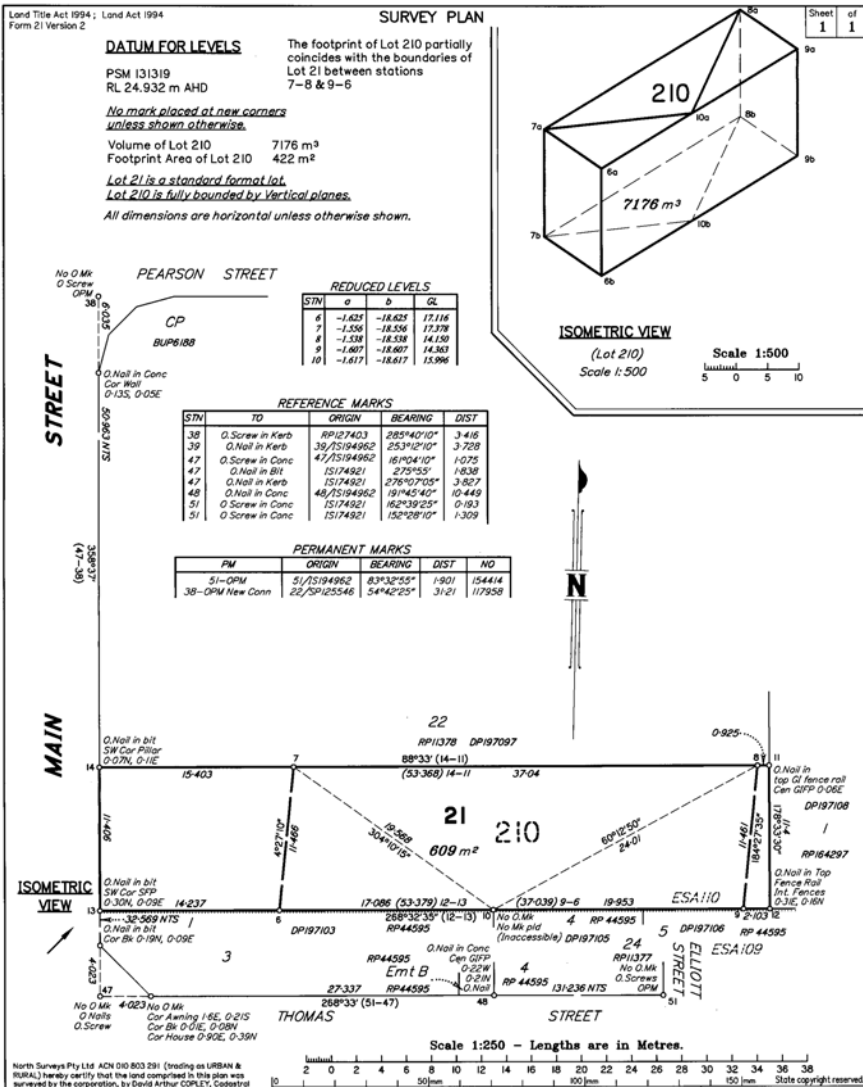


Unless a region is fully “closed” (isCycle),
its interior may not be well defined

Invalid data which looks 'correct'
(as in principle region is clear and
within tolerance)

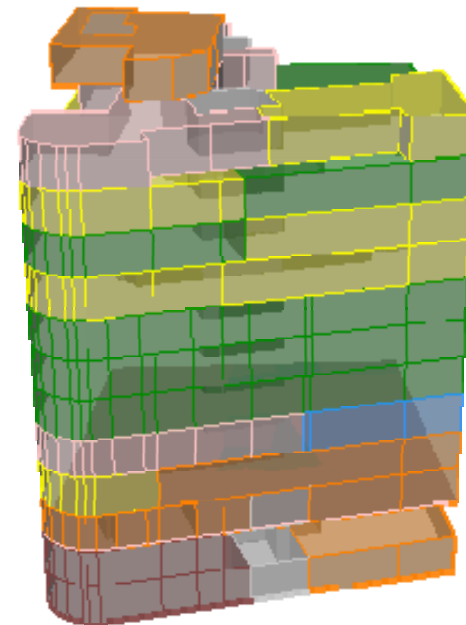


Bounded and unbounded parcels

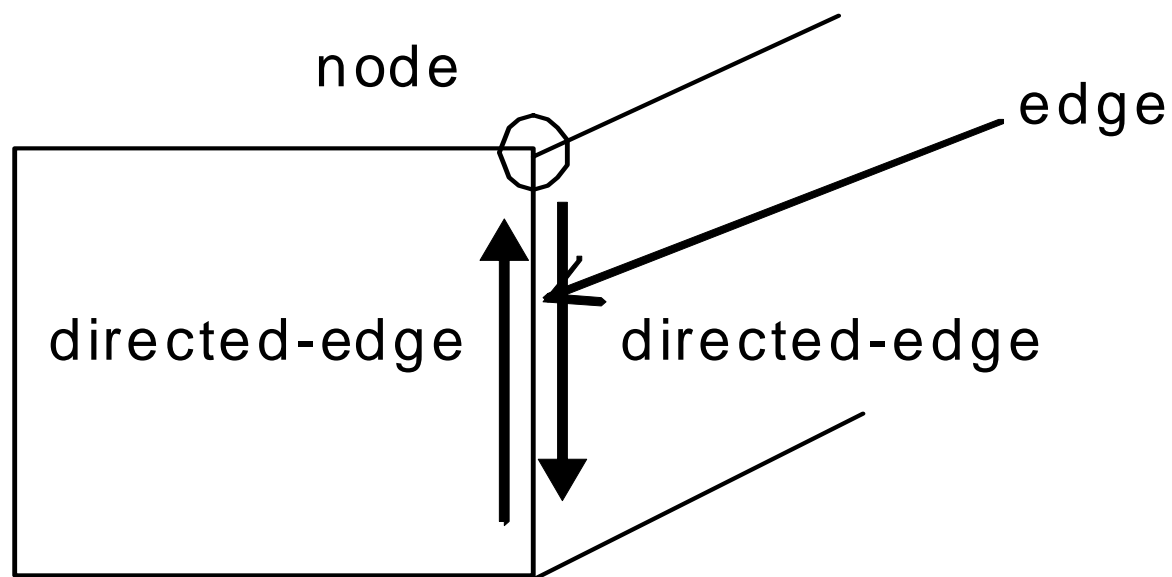


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Basic definitions

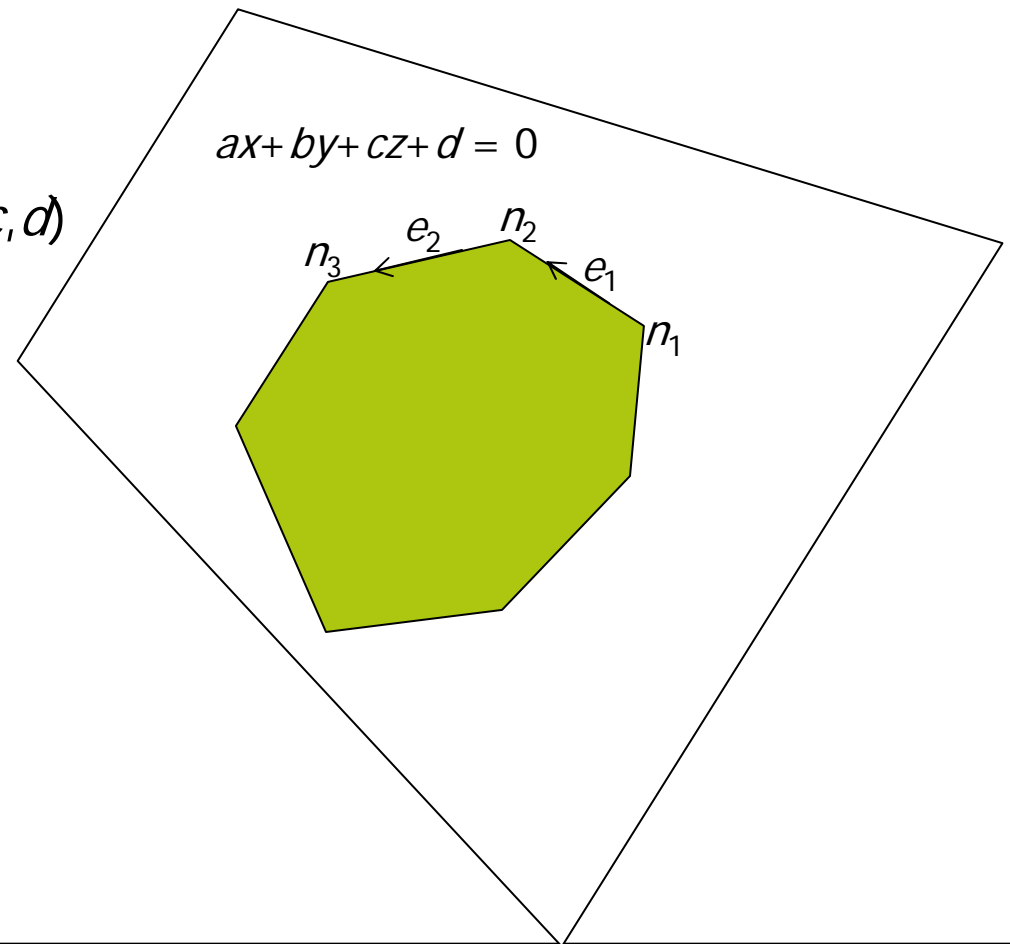


Basic Definition - Face

- A Face is defined by:
- a set of nodes f_n
- a set of directed-edges f_e
- a tuple of numbers $f_p = (a, b, c, d)$
(defining a plane)

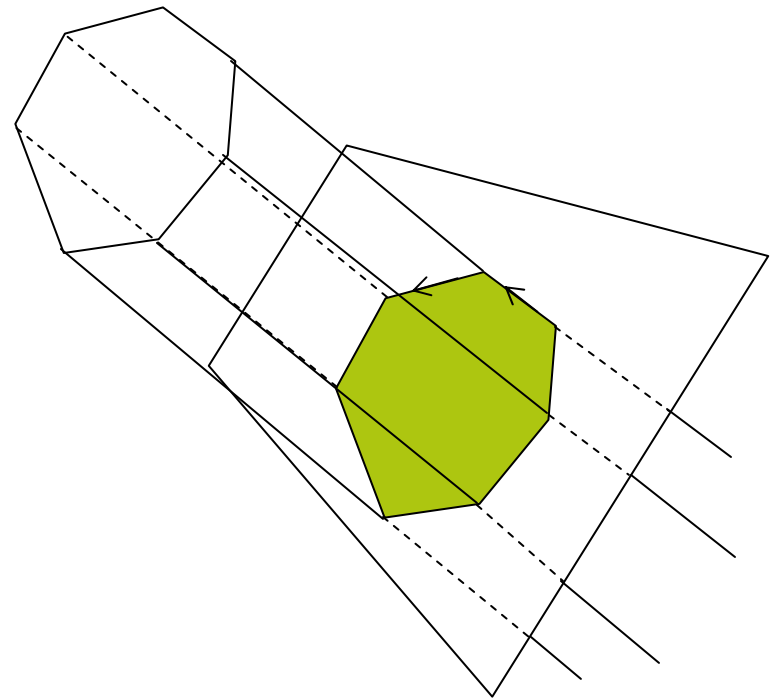
Note – that this is over-defined

Possibly not necessary to interchange the plane.

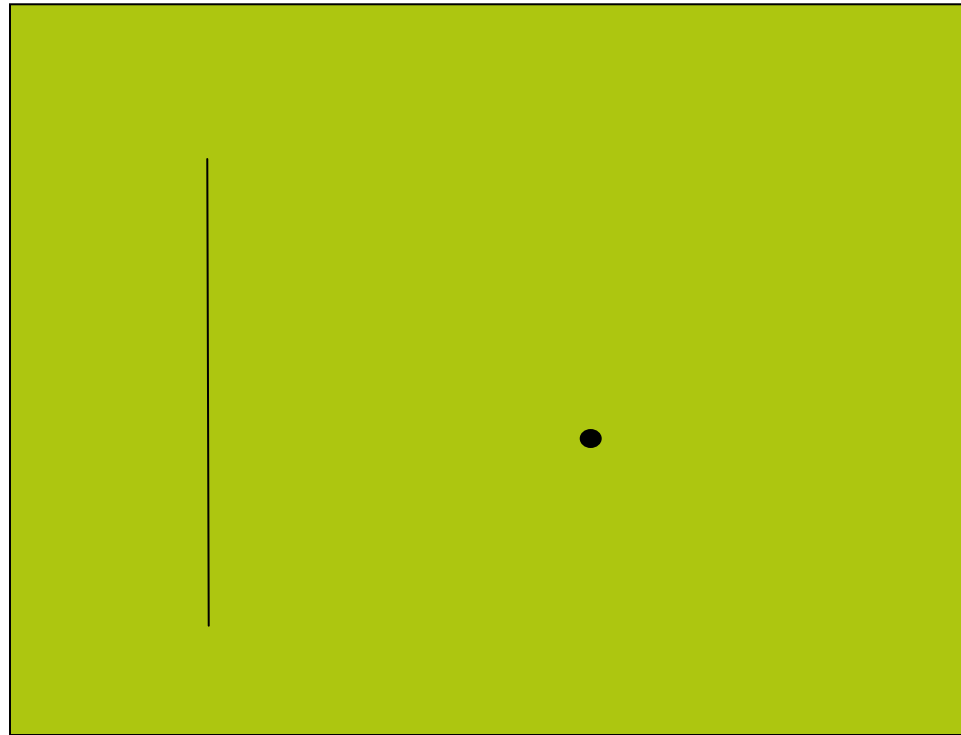


Basic Definition - Face

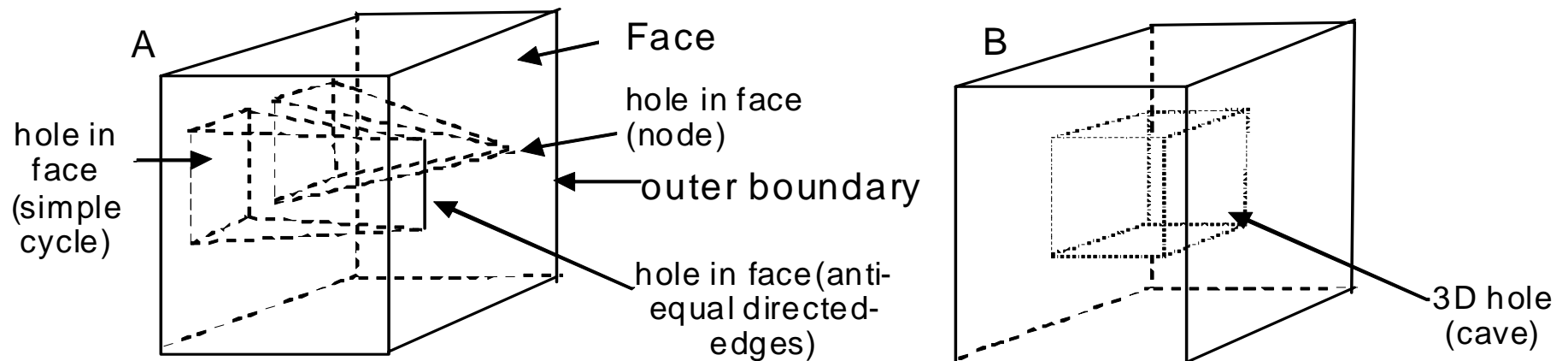
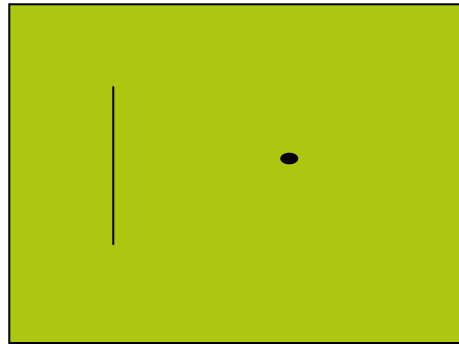
- Where two faces are defined on the same set of nodes, the plane parameters must agree.



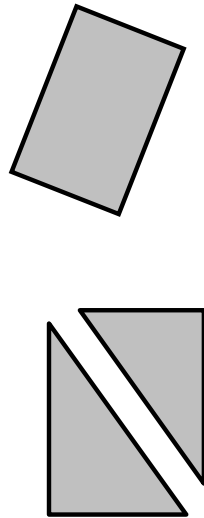
Holes in faces and in regions



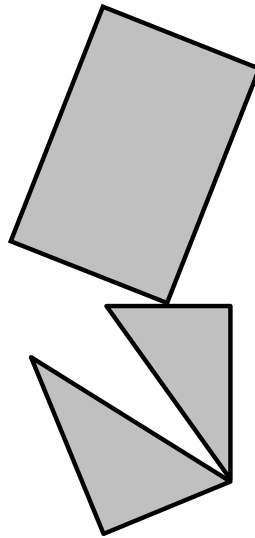
Holes in faces and in regions → various flavours



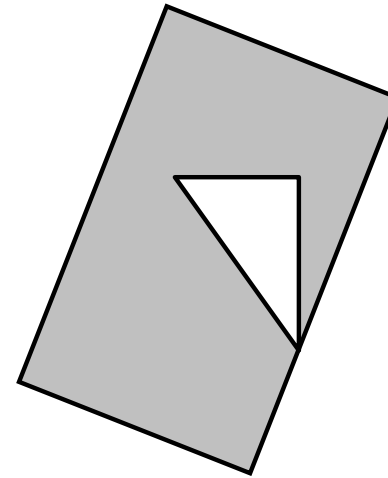
Connectivity - Faces



face

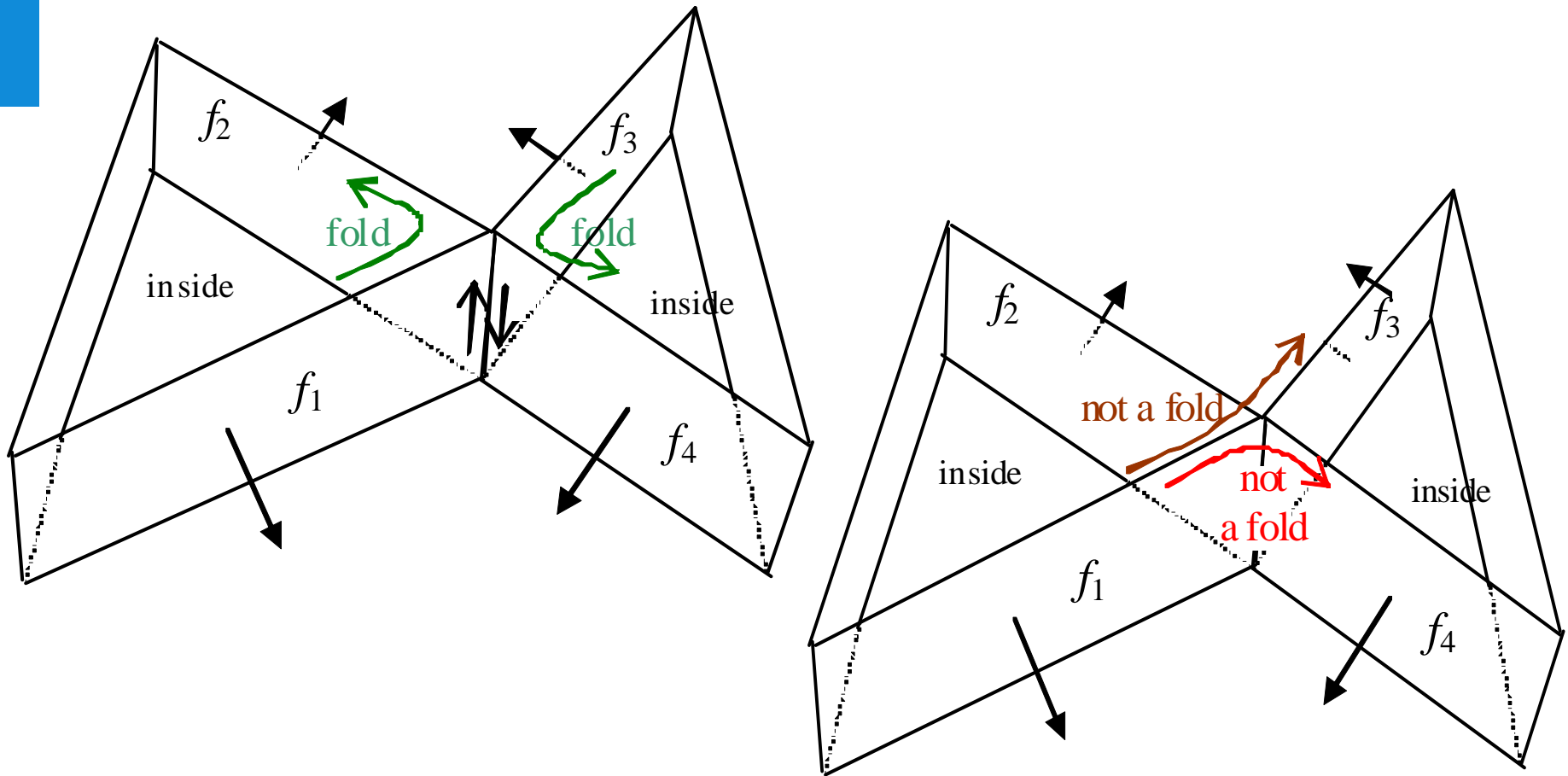


C_0 face

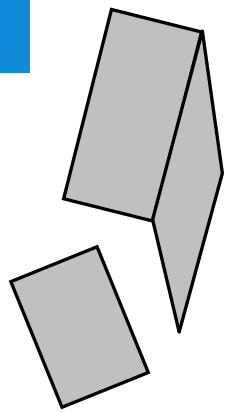


C_1 face

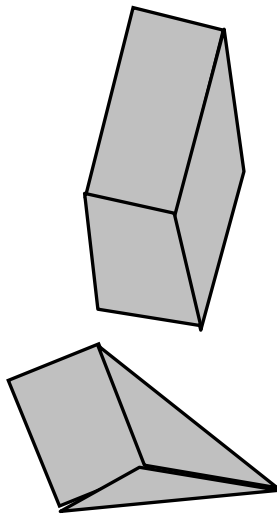
Connectivity of Faces (Fold)



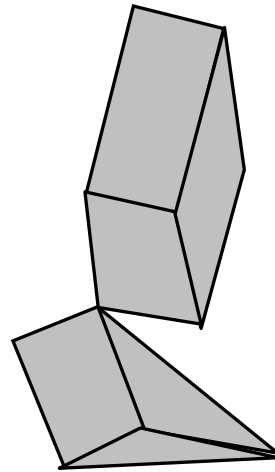
Connectivity - Shells



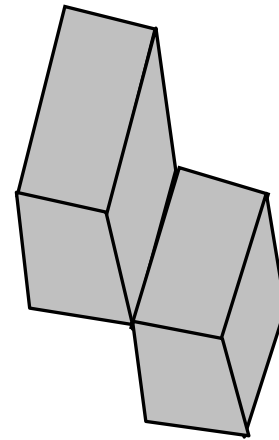
shell



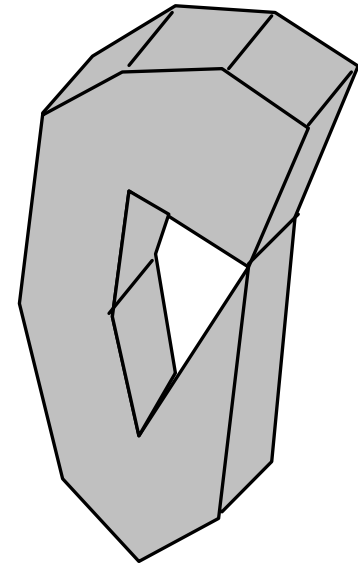
cycle shell



C₀ shell



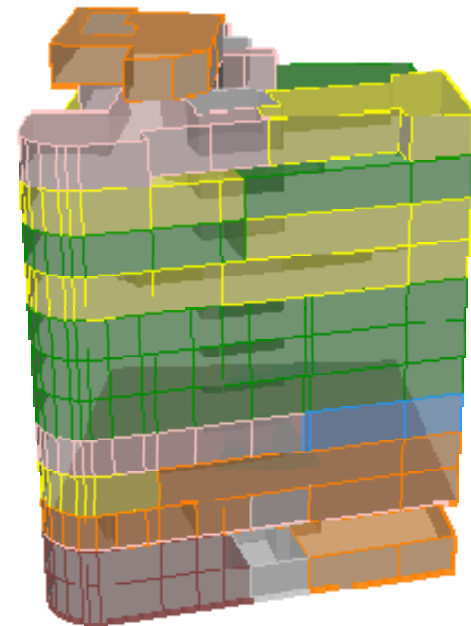
C₁ shell



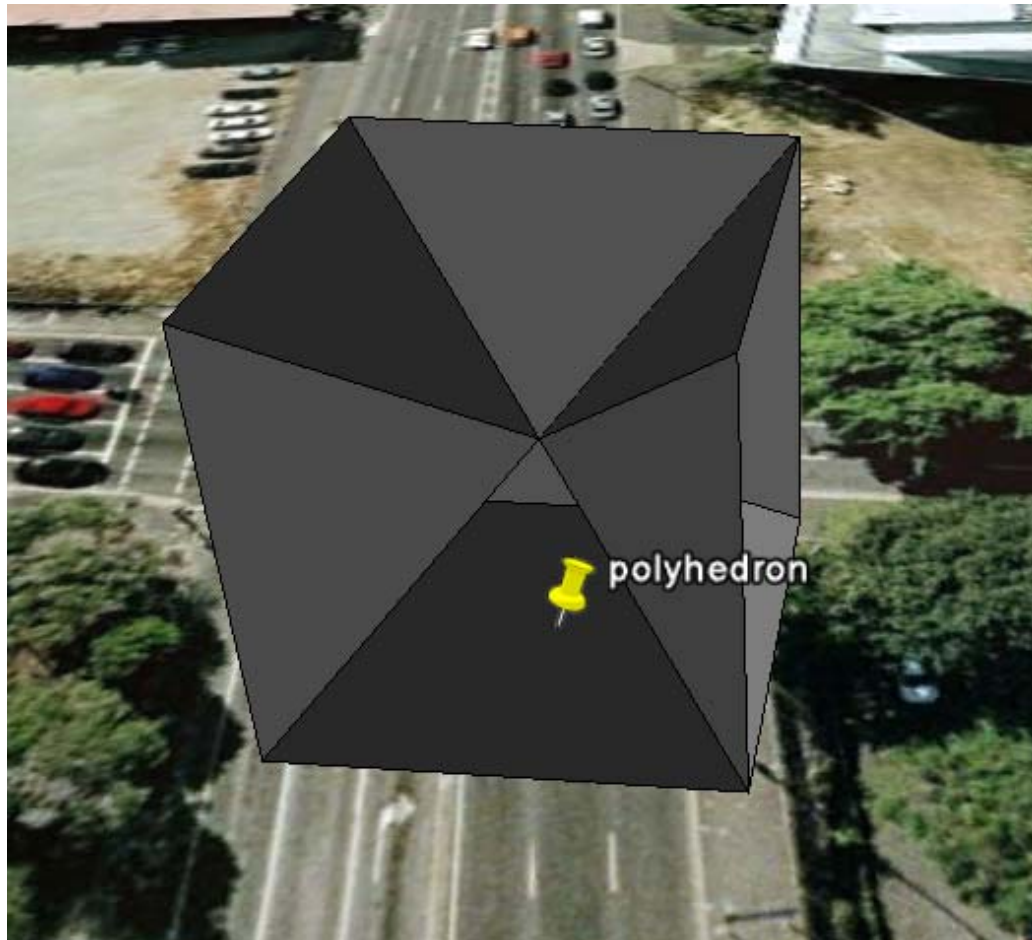
C₂ shell

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Axiom A1



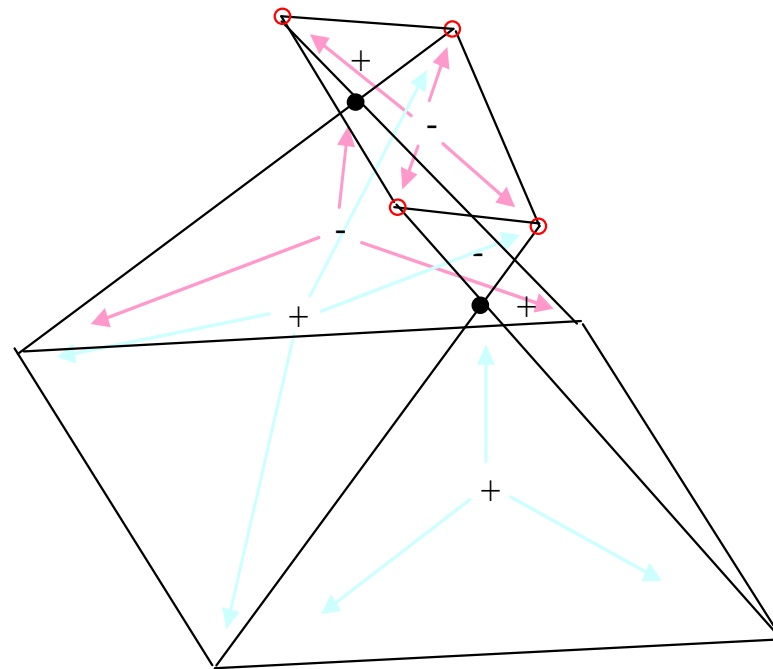
No two nodes are closer than ε apart

(figure looks inside volume by not showing top/front faces)

Axiom A3



The faces incident at a node do not intersect one another



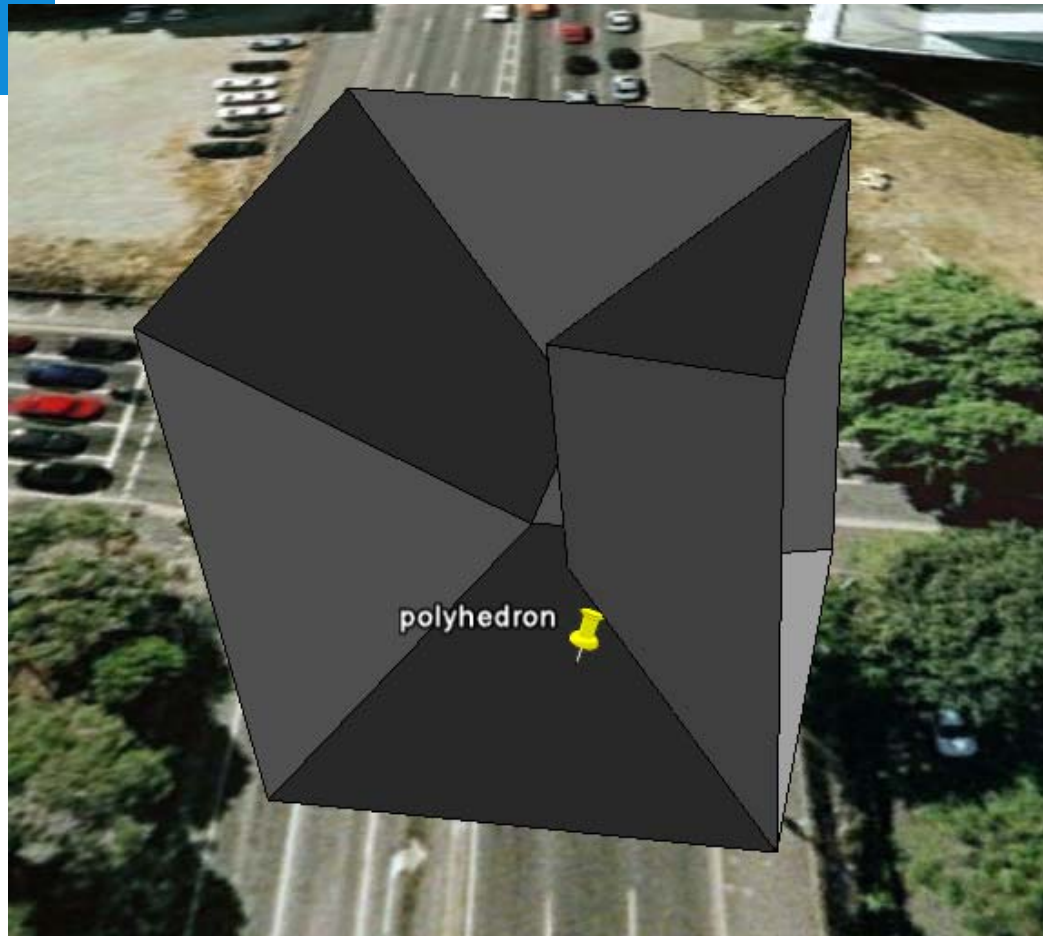
(note that involved edges have proper nodes at intersection)

Axiom A4

Edges do not intersect except at their common end point nodes

This axiom proved to be redundant (see A5)

Axiom A5

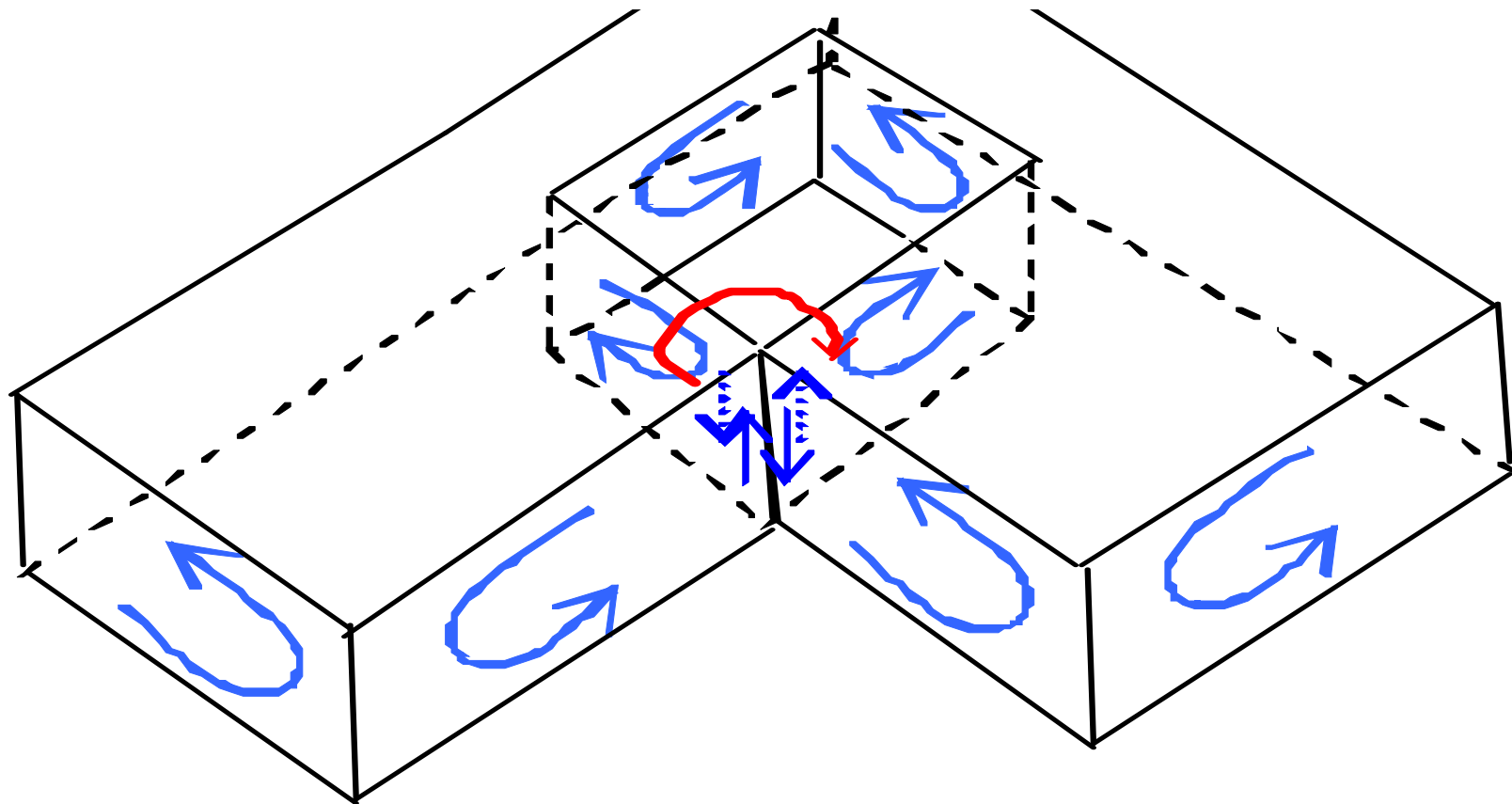


Non intersecting edges must not be within a distance ε of each other

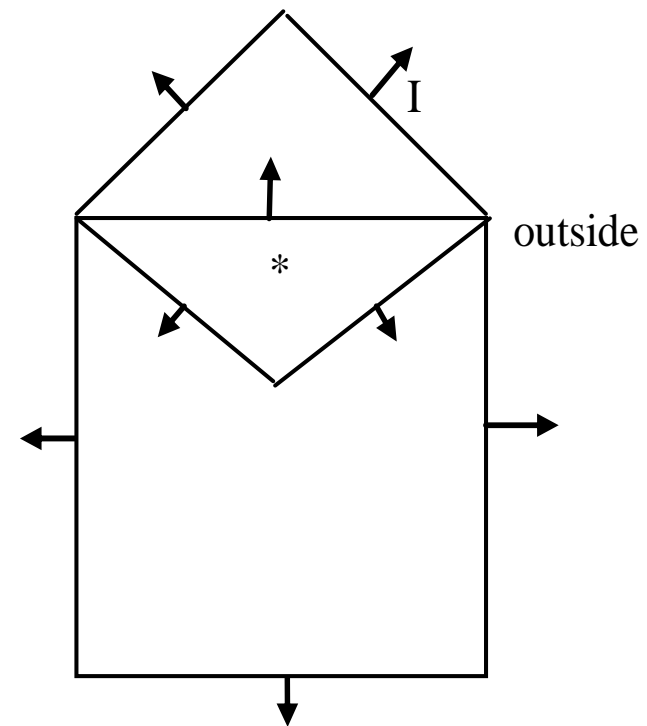
(note all nodes do pass Axiom A1: enough apart; top and front faces have been made transparent)

Axiom A6

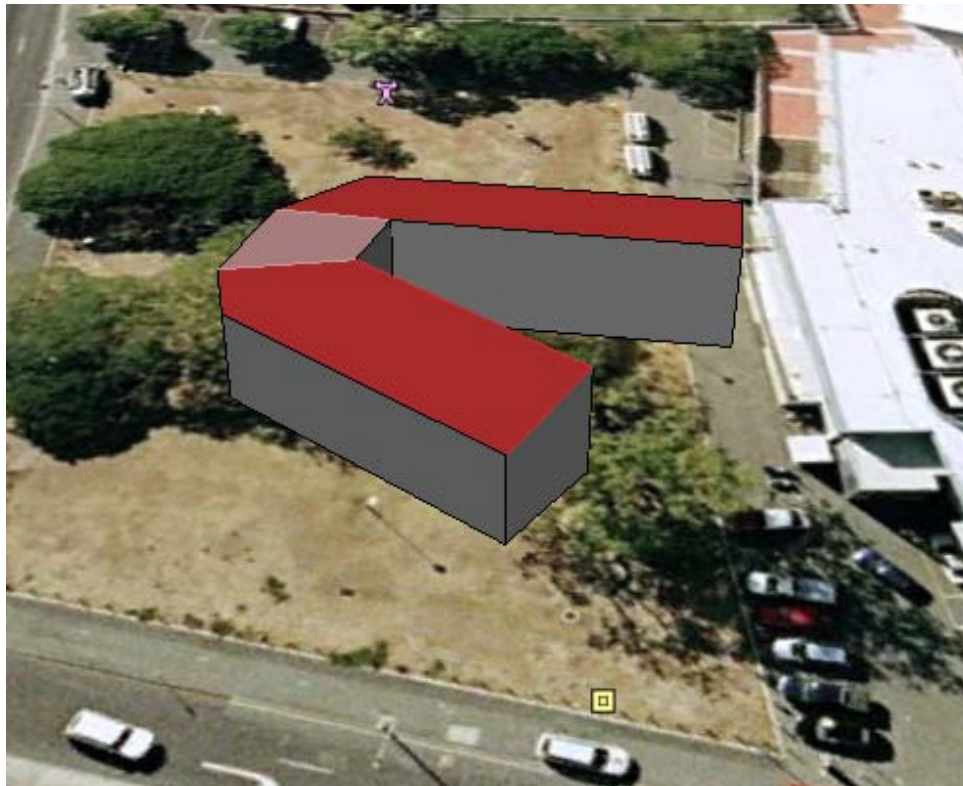
Each edge is the meeting of an even number of faces such that the directed-edges of those faces form an alternating sequence around the edge



Axiom A6 Example



Axiom A8

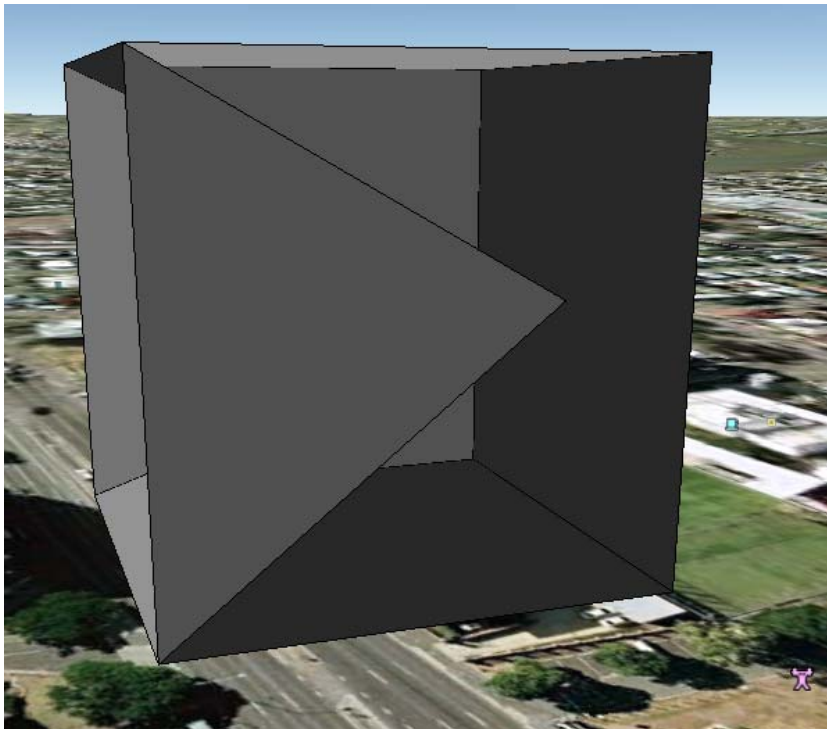


Bounded faces are planar to a tolerance of ε'

(given face defined by more than 3 points, this is non-trivial axiom)

Axiom A9

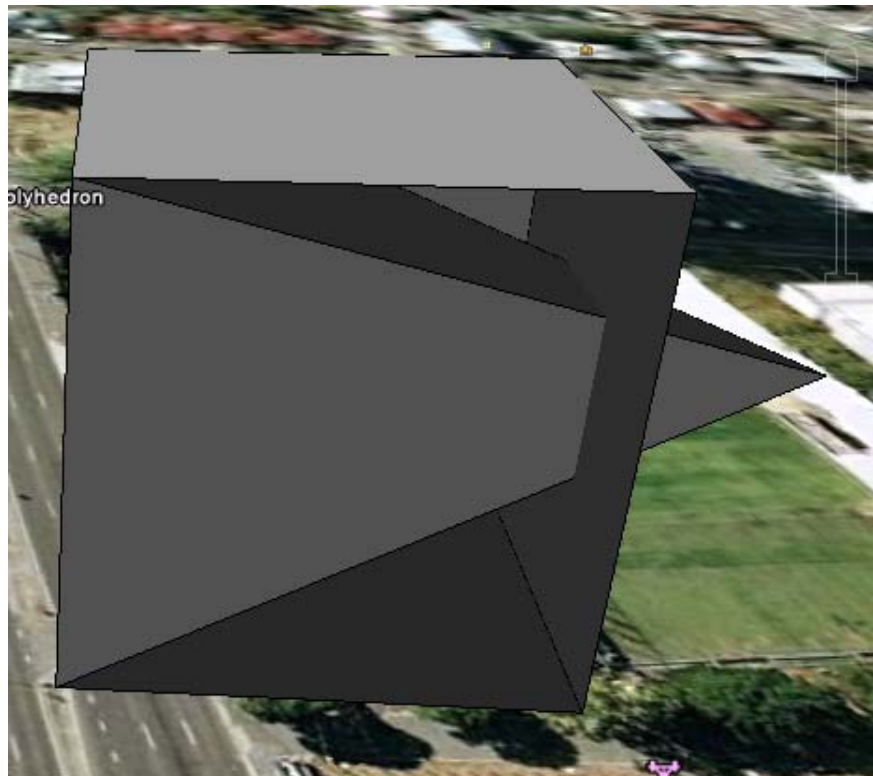
No node is within ϵ of a face unless it is part of the definition of that face



(front face omitted in figure)

Axiom A10

No edge intersects a face except at a node of that face

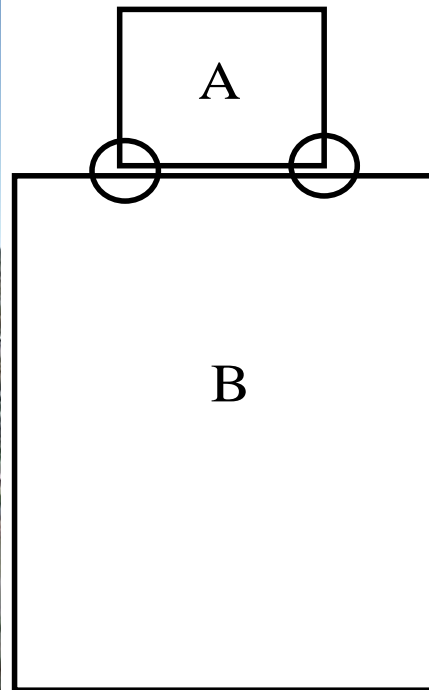
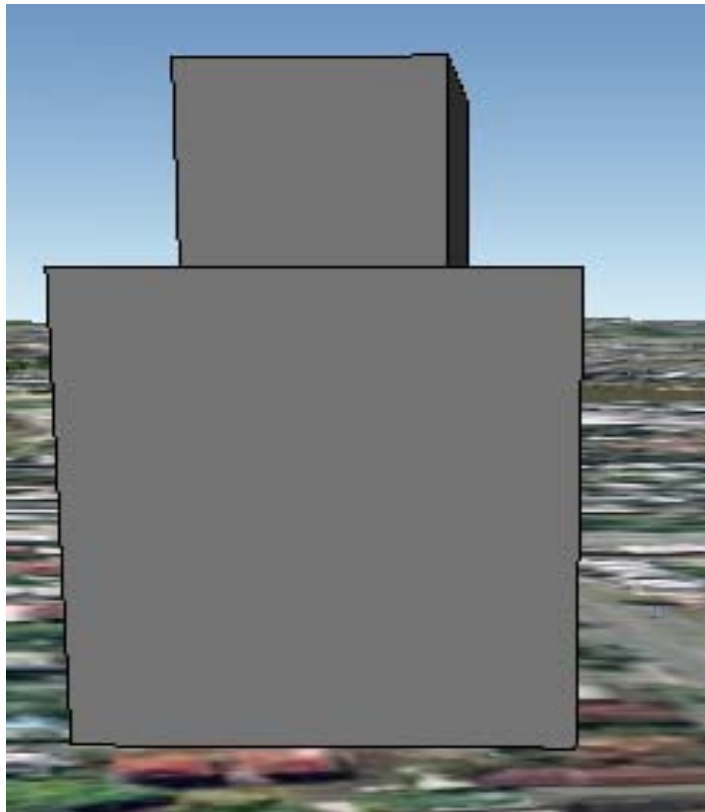


(note 1: front face omitted in figure to the left)

(note 2: Axiom A3 example has nodes where the faces cross → different case)

Axiom S1

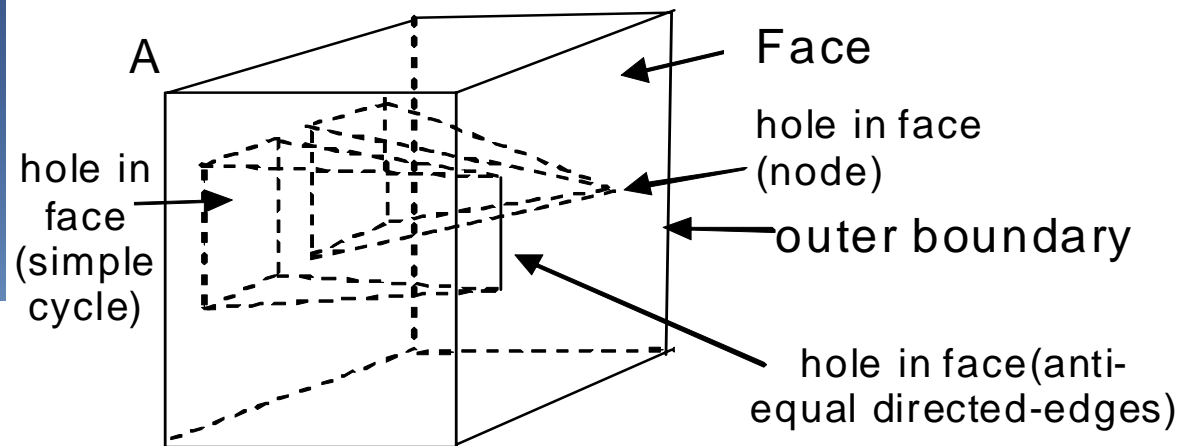
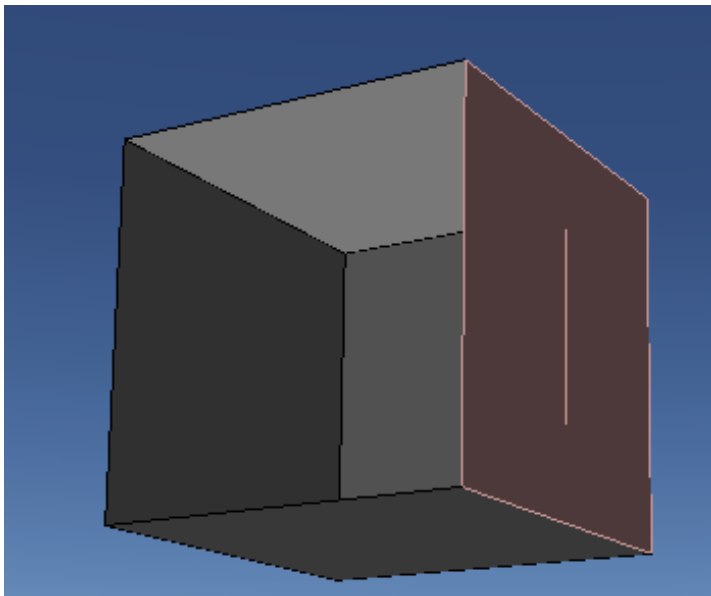
- No face may be paired with an anti-equal face in the same shell.



Spoils the definition of 2-shell

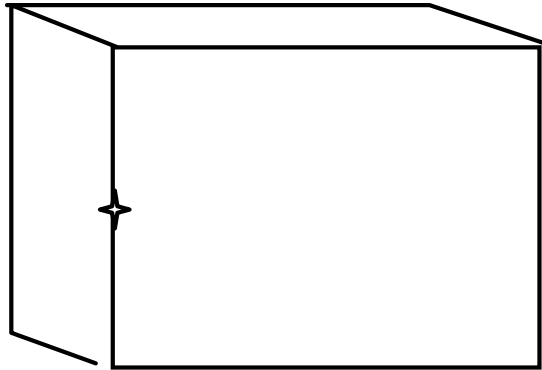
Axiom A7 (Optional)

For any node, there must be at least two distinct faces that it participates in.

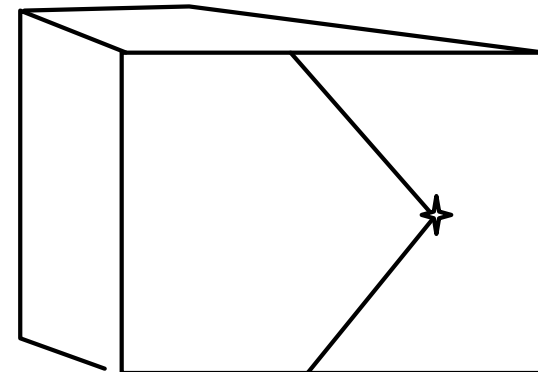


Axiom A2 (Optional)

Each node has at least 3 incident faces

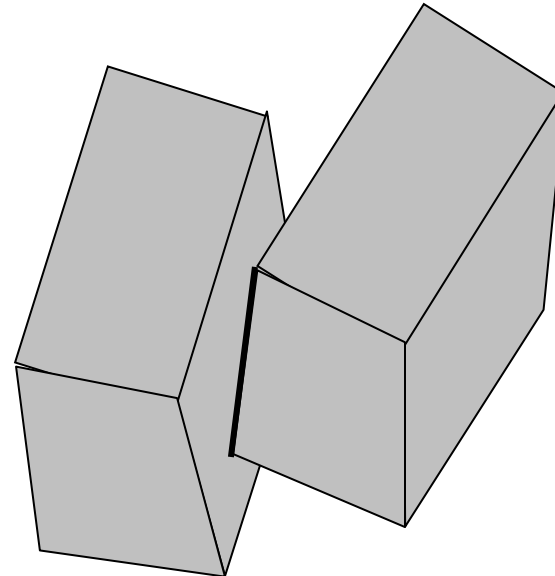
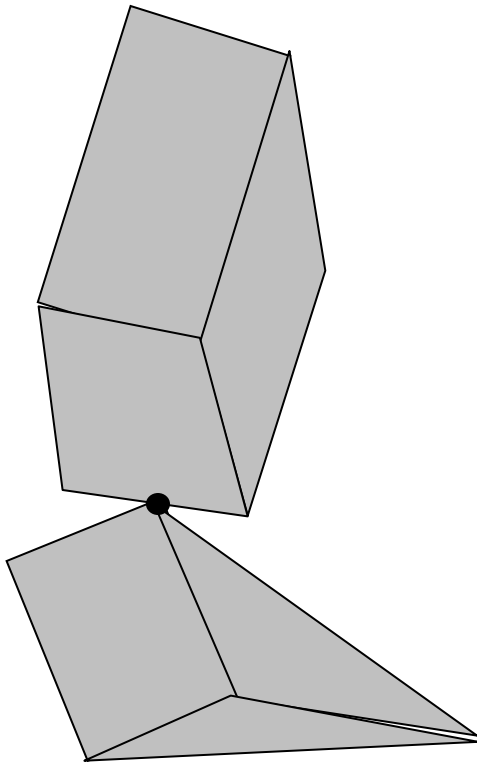


(marked node to be removed)

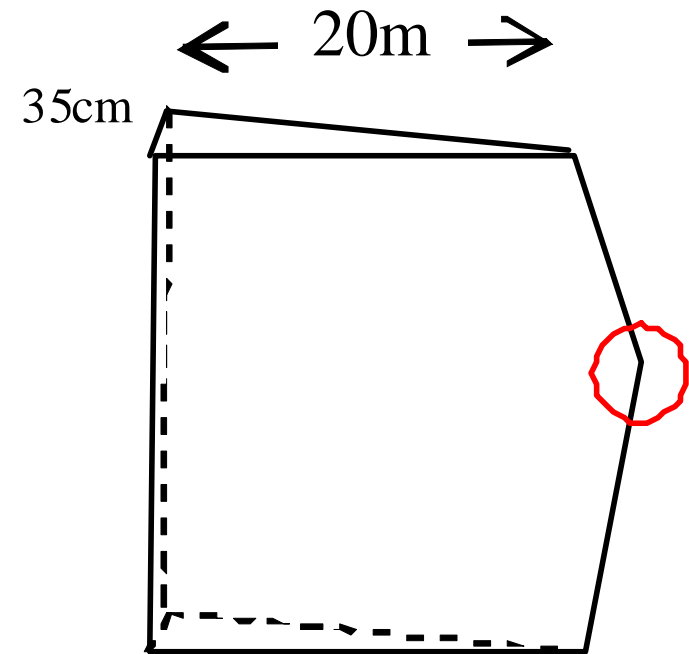
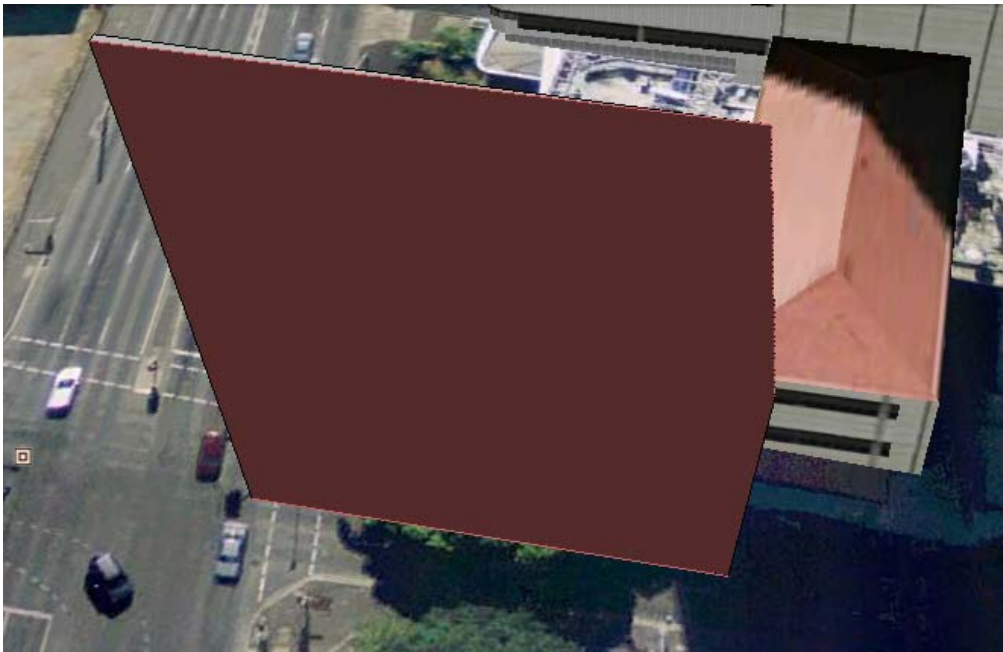


(marked node and incident edges to be removed)

Optional Axioms 2 and 7 (against)

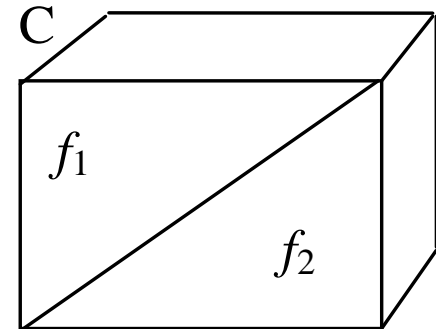
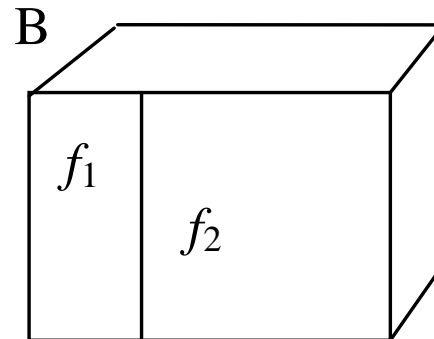
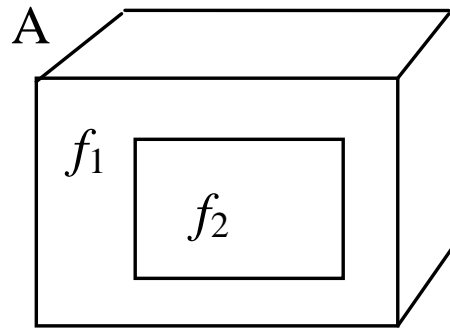


Optional Axiom 2 (for)

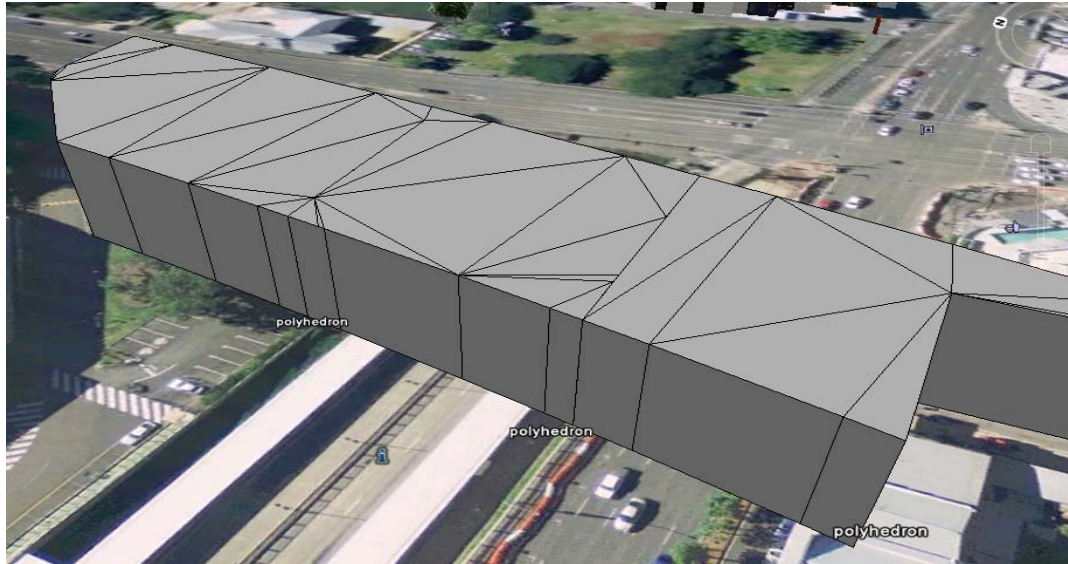


Faces are planar to within 5.6 mm

Other Possible Axioms



No neighbour faces in same plane.



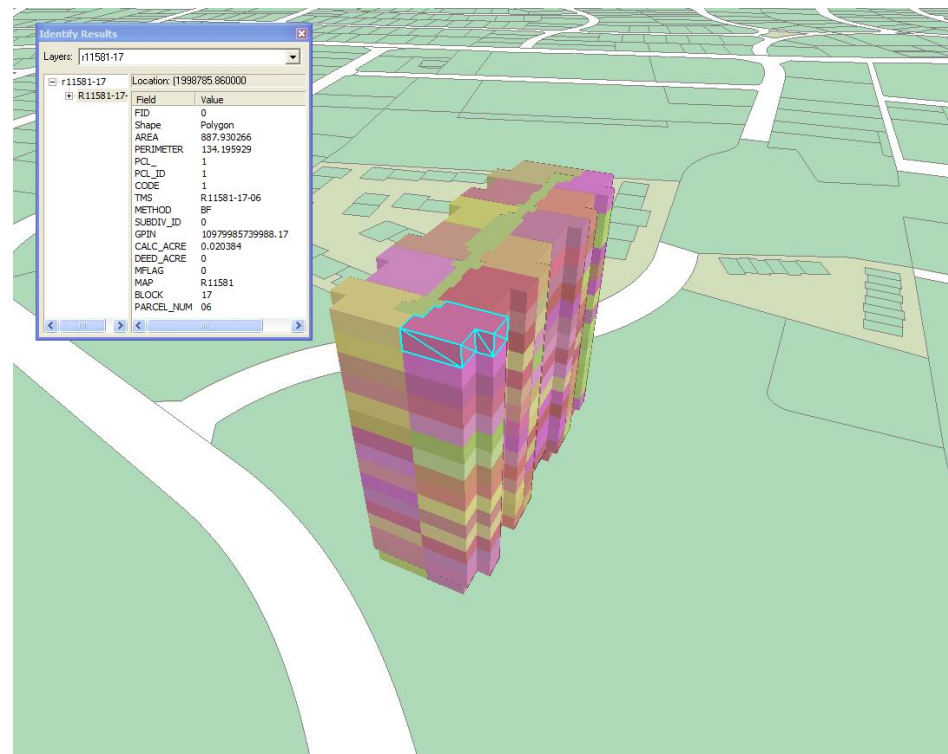
And why not.

Further refinements

- Axioms A1-A10 are base axioms
- Allow weakly connected/disconnected objects
- Additional axioms for 'flavours' of single parcels; e.g. Axiom S1. No face paired with anti-equal face in the same parcel (disallowing dangling double faces, and interior cut)
- Additional axioms for 'flavours' of space partitions; e.g. No point of parcel is inside another parcel or Each face paired with exactly 1 anti-equal face (at boundary of domain with faces of universal volume)

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Land Administration Domain Model (ISO 19152)

“Spatial Unit” maps to (probably) a C2 shell

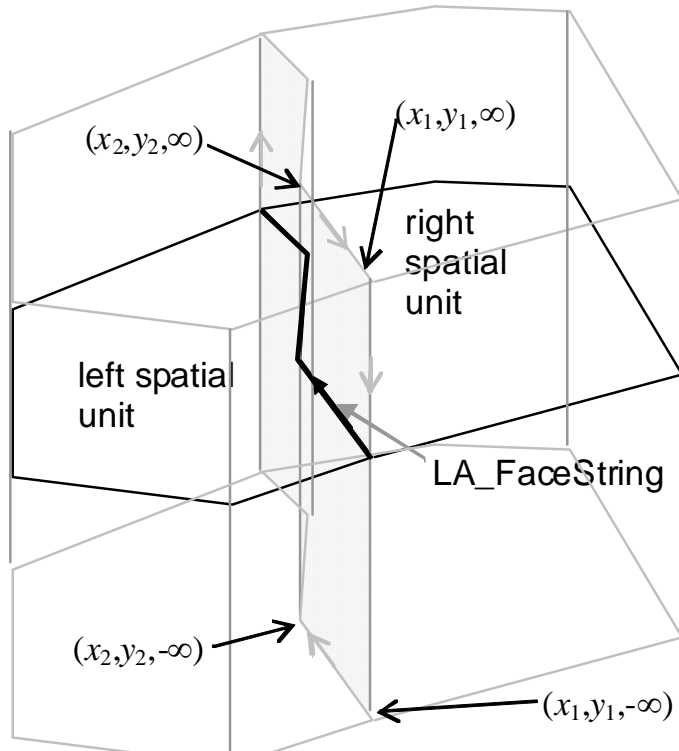
“Face String” maps to a set of faces

“Face” maps to a face or a pair of faces

Given these mappings, the axioms can be applied.

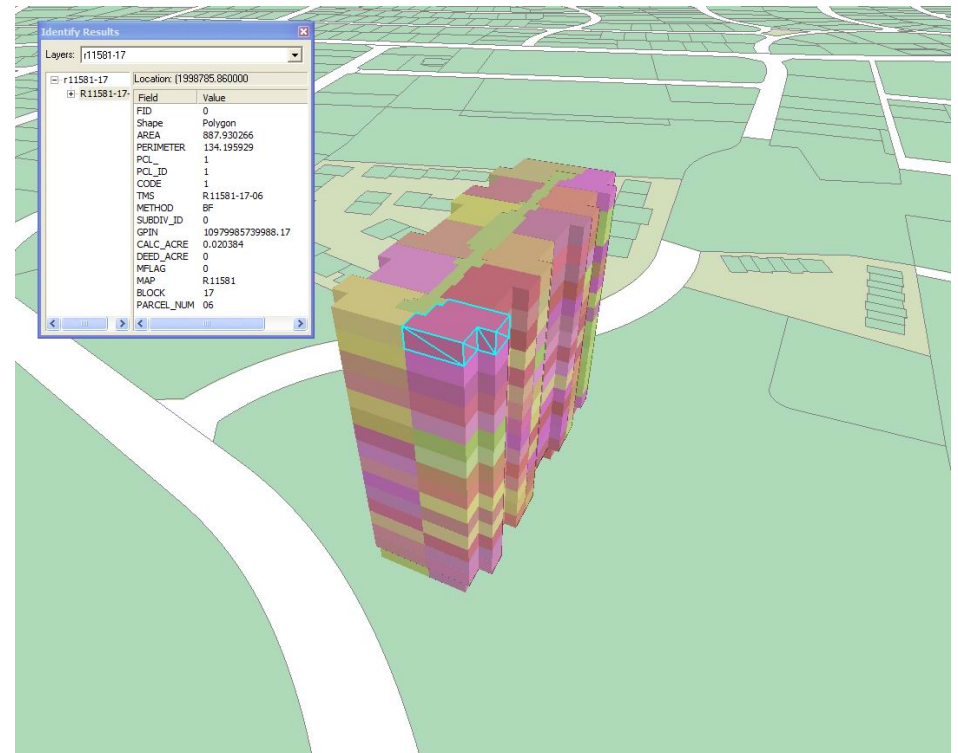
Topology-based storage possible

- Need concept of $Z = \pm\infty$



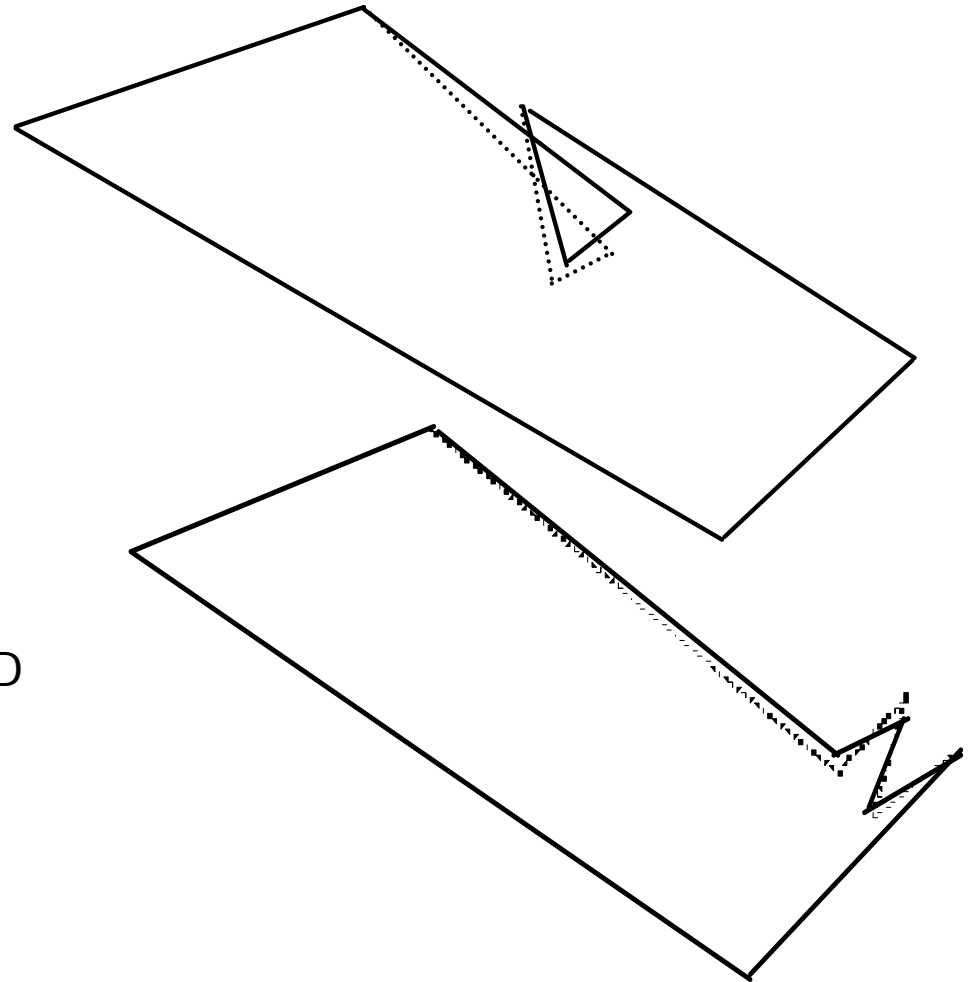
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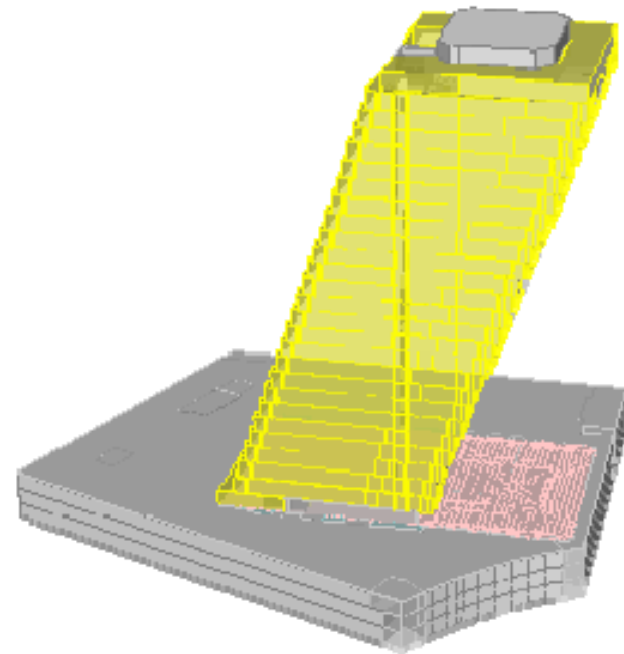
Invalid face?

- Interesting case reported by Coors, 2010 →
- All axioms passed, including:
 - Axiom A4. no 3D edge intersection
 - Axiom A8. face planar within ϵ'
- Still an unwanted situation
- Extension of a similar problem in 2D
- Axioms are not complete (in the sense that “bad things” can still happen).



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Conclusion

- Axioms: vendor independent, well documented 3D parcel definition
- Axioms are independent, non-redundant
- Axioms for individual parcels/space partitioning
- Modifications possible by:
 1. Making stricter by adding axioms (A7, A2)
 2. Making more forgiving by omitting axioms (S1)
- Result is taxonomy of families of valid 3D objects
- 3D parcels, non 2-manifold, partially unbounded,...
→ forwarded to ISO TC211 for revision of ISO 19107

Future work

- Extend to other representations (e.g. regular polytope)
- Formalise unbound parcel handling
- Prove additional consequences (e.g. cycle shell \equiv regular polytope)
- Further consideration of non-exact arithmetic
- Consider non-planar surfaces